

# Linear Model-Dependent Control

Model-dependent control for single-input/single-output discrete-time linear systems is introduced. It is a powerful controller design method that effectively handles open-loop instability, inverse response, and time delays. Only two parameters need to be tuned, and these directly influence control quality and robustness. Two examples demonstrate the power and versatility of the method.

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## Introduction

The majority of control loops are well behaved, and PID controllers perform satisfactorily. However, there are problem cases; for example, processes with inverse response and time delays. The recently developed internal model control (IMC) method (Garcia and Morari, 1982; 1985a, b; Morari, 1983; Morari et al., 1984; Economou et al., 1984; Brosilow and Tong, 1978; Brosilow, 1979; Chen and Brosilow, 1984; Zames, 1981; Desoer and Chen, 1981) provides an easy yet effective way to deal with such cases. IMC essentially recasts the feedback design problem into a feedforward problem. The usual IMC design procedure consists of factoring the plant transfer function into an invertible part and a noninvertible part, and then choosing the IMC controller as the inverse of the invertible part with filtering included to improve robustness. This simple and elegant procedure gives excellent control for set point changes and also does quite well in rejecting disturbances if the open-loop disturbance dynamics are not slow. Therefore it can also be used to tune PID controllers (Morari et al., 1984). However, this inversion-based IMC design procedure will not give satisfactory disturbance rejection if the disturbance dynamics are very slow. (For this reason Morari and Zafiriou, 1986, advocate an alternate procedure for disturbance rejection.)

Model-dependent control (MDC) is a simple method which transforms the problem of designing a controller that effectively handles both set point changes and disturbances (including those with slow dynamics) to that of designing an IMC controller using the usual inversion-based procedure. An additional advantage is that MDC does not need open-loop stability, a requirement of the usual inversion-based IMC design procedure.

## Analysis of Inversion-Based IMC

If a single-input/single-output linear process is directly computer-controlled, it can be described by a discrete-time linear model (Garcia and Morari, 1982; Stephanopoulos, 1984). This

in turn has the  $z$  domain representation

$$\bar{y}(z) = G_u(z)\bar{u}(z) + \sum_{i=1}^r G_i(z)\bar{d}_i(z) \quad (1)$$

where  $\bar{y}(z)$ ,  $\bar{u}(z)$ ,  $\bar{d}_i(z)$  are respectively the  $z$  transforms of the measured variable, the control variable, and the  $i$ th disturbance.  $G_u(z)$  and  $G_i(z)$  are respectively the plant and  $i$ th disturbance pulse transfer functions.

In IMC the classical feedback structure of Figure 1 is recast into the IMC structure of Figure 2. Given open-loop stability, the two structures are equivalent provided

$$G_I = \frac{G_c}{1 + G_c\hat{G}_u} \leftrightarrow G_c = \frac{G_I}{1 - \hat{G}_u G_I} \quad (2)$$

Here  $G_I$  is the IMC controller,  $G_c$  the classical controller, and  $\hat{G}_u$  the available plant model.

Recognizing that  $\hat{G}_u^{-1}$  would be the perfect IMC controller (it gives  $\bar{y} = \bar{y}_{sp}$ ) if implementable, the controller is usually chosen as

$$G_I(z) = \hat{G}_u(z)^{-1}F(z); \quad F(1) = 1 \quad (3)$$

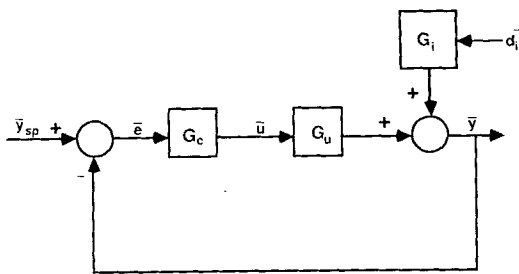
where the "filter"  $F(z)$  must:

- Make  $G_I$  realizable (cancel out the time delay in  $\hat{G}_u$ )
- Make  $G_I$  stable (cancel out the zeroes of  $\hat{G}_u$  outside the open unit circle). To avoid ringing, zeroes with negative real part may also be canceled.
- Provide low pass filtering to increase robustness. This part includes a free parameter  $\epsilon$ . It usually is  $(1 - \epsilon)/(1 - \epsilon z^{-1})$  with  $\epsilon \in [0, 1)$ .

More information about the choice of  $F(z)$  can be found in Zafiriou and Morari (1985). For example, if no zeroes need be canceled the filter is typically chosen as

$$F(z) = \frac{(1 - \epsilon)z^{-k-1}}{1 - \epsilon z^{-1}} \quad (4)$$

where  $(k + 1)$  is the plant model time delay.



**Figure 1. Classical feedback control structure.**

$G_c$ , controller;  $G_u$ , plant;  $y_{sp}$ , set point;  $y$ , output;  $u$ , control;  $d_i$ , disturbances

For a perfect model ( $\hat{G}_u = G_u$ ) the resulting closed-loop equation is

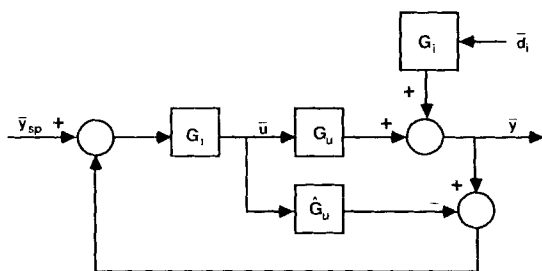
$$\bar{y}(z) = F(z)\bar{y}_{sp}(z) + [1 - F(z)] \sum_{i=1}^r G_i(z)\bar{d}_i(z) \quad (5)$$

It is seen that the response to set point changes involves only the filter pole(s). By tuning the filter one can achieve the desired process behavior and can theoretically obtain close to perfect response. However, disturbance response involves the disturbance transfer function poles in addition to the filter poles. If one of the disturbance poles is close to the unit circle then disturbance rejection will be slow. Furthermore, the system might deviate significantly from the desired steady state before eventually returning to it (and will do so only if the system is open-loop stable).

From one point of view, this simple IMC design procedure approximates as well as possible the inversion of the model (perfect control). From another point of view, it is a control scheme in which the closed-loop poles are not moved from their original open-loop location and additional pole(s) are added through the filter. For set point changes the open-loop poles are cancelled (approximately if the model is not exact), but for disturbance changes no cancellation occurs. Thus this controller requires open-loop stability and responds poorly to disturbances if the open-loop dynamics are slow.

### The Model-Dependent Control Law

The above analysis shows that if we could move the open-loop poles closer to the origin, we would greatly improve the IMC performance. Since open-loop poles cannot be moved, the basic idea behind MDC is to define a function of the control  $u$  and the output  $y$  as a generalized input  $U$ ; this will be selected so that the "open loop" poles with respect to it are in desired locations. Sub-



**Figure 2. Internal model control (IMC) structure.**

$\hat{G}_u$ , plant model

sequently the generalized input will be designed as an inversion-based IMC controller. The present development is well suited for the case where  $\epsilon$  is the only IMC filter pole. In case it is not, better results can be obtained if MDC is slightly modified as prescribed in Papadoulis (1986) and in a forthcoming paper (Papadoulis et al., 1987).

The process model can be rewritten as

$$A(z)\bar{y}(z) = B_+(z)B_-(z)\bar{u}(z) + \sum_{i=1}^r C_i(z)\bar{d}_i(z) \quad (6)$$

where

$$A(z) = \sum_{i=0}^{n_a} a_i z^{-i}, \quad a_0 = 1 \quad (7)$$

$$B_+(z) = z^{-k-1} \sum_{i=0}^{n_+} \beta_i z^{-i}, \quad \beta_0 \neq 0 \quad (8)$$

[ $B_+(z)$  contains the noninvertible elements of  $\hat{G}_u(z)$ .]

$$B_-(z) = \sum_{i=0}^{n_-} b_i z^{-i}, \quad b_0 \neq 0 \quad (9)$$

[All zeroes of  $B_-(z)$  are inside the open unit circle.]

$$C_j(z) = \sum_{i=0}^{n_j} c_{ji} z^{-i} \quad (10)$$

There is no loss of generality in going from Eq. 1 to Eq. 6 since any disturbance poles that do not appear in the plant transfer function are added to the latter as pole/zero pairs. Consequently,  $A(z)$  is the least common denominator of  $\hat{G}_u$ ,  $\hat{G}_1$ , ...,  $\hat{G}_r$ . For example

$$\bar{y} \approx \frac{z^{-1}}{1 - 0.5z^{-1}} \bar{u} + \frac{z^{-1}}{1 - 0.9z^{-1}} \bar{d}$$

can be rewritten in the form of Eq. 6 with

$$A(z) = 1 - 1.4z^{-1} + 0.45z^{-2}$$

$$B_+(z) = z^{-1}$$

$$B_-(z) = 1 - 0.9z^{-1}$$

$$C(z) = z^{-1} - 0.5z^{-2}$$

Consider now the generalized input

$$\bar{U}(z) = Q(z)B_-(z)\bar{u}(z) + P(z)\bar{y}(z) \quad (11)$$

where

$$P(z) = \sum_{i=0}^{n_p} p_i z^{-i} \quad (12)$$

$$Q(z) = \sum_{i=0}^{n_q} q_i z^{-i}, \quad q_0 = 1 \quad (13)$$

Equation 6 can be rewritten in terms of this generalized input as

$$[A(z)Q(z) + B_+(z)P(z)]\bar{y}(z) = B_+(z)\bar{U}(z) + Q(z) \sum_{i=1}^r C_i(z)\bar{d}_i(z) \quad (14)$$

Thus the "open loop" poles are now the zeroes of

$$G(z) = A(z)Q(z) + B_+(z)P(z) \quad (15)$$

If the generalized input is designed as an IMC controller, we have

$$\bar{U} = \frac{G_I}{1 - \hat{G}_U G_I} \bar{e}; \quad \hat{G}_U = \frac{B_+}{G}; \quad G_I = GM \quad (16)$$

where  $M = F/B_+$  with the pole/zero pairs canceled (i.e.,  $M$  has no unstable poles and no prediction elements).

The above controller gives the closed-loop system equation

$$\bar{y}(z) = F(z)\bar{y}_{sp}(z) + \frac{[1 - F(z)]Q(z)}{G(z)} \sum_{i=1}^r C_i(z)\bar{d}_i(z) \quad (17)$$

The closed-loop response depends on the filter pole(s) and the zeroes of  $G(z)$ . Letting

$$G(z) = \sum_{i=0}^{n_a} \alpha^i a_i z^{-i} \quad (18)$$

sets the zeroes of  $G(z)$  at

$$z_i = \alpha \lambda_i \quad i = 1, \dots, n_a \quad (19)$$

where  $\lambda_i$  are the open-loop eigenvalues of system 6. As  $\alpha$  decreases from 1 to 0 the zeroes of  $G(z)$  shrink toward the origin, thus speeding up the closed-loop dynamics.

Even if the system is open-loop unstable there is an  $\alpha_{max}$  such that for  $\alpha \leq \alpha_{max} |\lambda_i| < 1$ ;  $i = 1, \dots, n_a$ . Let us assume that for  $\alpha = \alpha_{max}$  the closed-loop system is robust (stable in spite of plant/model mismatch); if the system is open-loop stable this assumption is easily satisfied since for  $\alpha = \alpha_{max} = 1$ ,  $G(z) = A(z)$  and  $u = U$ , which was designed via the conventional IMC methodology (where robustness is guaranteed for a high enough choice of the filter pole  $\epsilon$ ). Parameter  $\alpha$  can then be viewed as a parameter that adjusts the trade-off between robustness and speed of response. For  $\alpha$  close to  $\alpha_{max}$  we have robustness, and for  $\alpha = 0$  we have maximum speed. However, as  $\alpha$  decreases robustness may be lost; see example 2, below.

Equation 18 requires that we choose  $P(z)$  and  $Q(z)$  as solutions of the Diophantine equation

$$A(z)Q(z) + B_+(z)P(z) = \sum_{i=0}^{n_a} \alpha^i a_i z^{-i} \quad (20)$$

This is solvable provided  $B_+(z)$  and  $A(z)$  do not have a common zero (in which case the system is not stabilizable). In the general case it has infinite solutions. However, a unique minimal-order solution is obtained by setting the degrees of  $P(z)$  and  $Q(z)$  to

$$n_p = n_a - 1 \quad (21)$$

$$n_q = k + n_+ \quad (22)$$

Bringing everything together, the MDC controller is obtained as follows:

1. Calculate the unique  $P(z)$  and  $Q(z)$  that satisfy Eq. 20 with order parameters as in Eqs. 21 and 22. These will be functions of  $\alpha$ .

2. Choose an IMC filter  $F(z) = B_+(z)M(z)$  for system 14. This version of MDC performs best for disturbance if  $\epsilon$  is the only filter pole.

3. Obtain the MDC law as:

$$QB_-(1 - F)\bar{u} = GM\bar{e} - P(1 - F)\bar{y} \quad (23)$$

(This was derived using Eqs. 16 and 11.)

4. Tune the parameters  $\epsilon$  and  $\alpha$  (see also the remark below). A block diagram of the MDC scheme is given in Figure 3.

**Remark.** It would be helpful if the robustness/speed of response trade-off were controlled through only one parameter. If the filter  $F$  is chosen so that  $\epsilon$  is its only pole, as is often the case, then reducing  $\alpha$  beyond the point where  $\epsilon$  is the dominant closed-loop pole will not, in most cases, result in large improvements of performance. Thus we can set  $\alpha = \min\{1, \epsilon/\max\{|\lambda_i|\}\}$ , reducing the number of parameters to be tuned to a single "robustness parameter"  $\epsilon \in [0, 1)$ , which can be easily adjusted on-line.

## Properties of MDC

**Property 1. Stability.** For small modeling errors a stabilizable linear plant controlled by the MDC law, Eq. 23, is closed-loop stable.

**Proof.** Let the true process, indicated by  $\sim$ , be

$$\tilde{A}\bar{y} = \tilde{B}_+\tilde{B}_-\bar{u} + \sum \tilde{C}_i\bar{d}_i \quad (24)$$

and define the errors

$$\Delta_A = \tilde{A} - A$$

$$\Delta_B = \tilde{B}_+\tilde{B}_- - B_+B_-$$

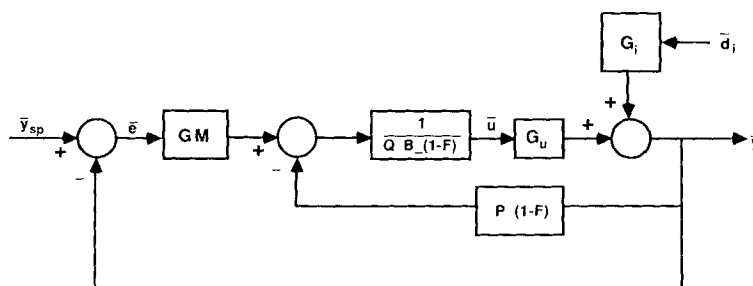


Figure 3. Model-dependent control (MDC) structure.

Then Eq. 24 and the control law, Eq. 23, give:

$$H\bar{u} = \tilde{A}GM\bar{y}_{sp} - (P + AQM)\Sigma\tilde{C}_i\bar{d}_i \quad (25)$$

$$H\bar{y} = \tilde{B}_+\tilde{B}_-GM\bar{y}_{sp} + QB_-(1-F)\Sigma\tilde{C}_i\bar{d}_i \quad (26)$$

where

$$H = GB_- + (P + AQM)\Delta_B + QB_-(1-F)\Delta_A$$

Now let  $w = z^{-1}$ . If  $|\Delta_A(w)|$  and  $|\Delta_B(w)|$  are small at each  $w$  on the unit circle, then by Rouché's theorem (Weinberger, 1965)  $H$  and  $GB_-$  have the same number of zeroes inside the unit circle (with  $w$  as the variable), or equivalently, the same number of zeroes outside the unit circle with  $z$  as the variable. Since  $GB_-$  has no such zeroes closed-loop stability follows.

**Remark.** Rouché's theorem can be used to prove that a sufficient condition for stability is  $|P + AQM||\Delta_B| + |QB_-(1-F)||\Delta_A| < |GB_-|$  for all  $w$  on the unit circle.

**Property 2. Pole Placement.** Assume that the model is exact. Then the closed-loop poles are the zeroes of  $G(z)$  and the filter pole(s).

**Proof.** If the model is exact

$$\Delta_A = \Delta_B = 0$$

$$\tilde{B}_- = B_-$$

$$\tilde{B}_+M = F$$

Thus Eq. 26 becomes

$$G\bar{y} = GF\bar{y}_{sp} + Q(1-F)\Sigma\tilde{C}_i\bar{d}_i$$

which proves the above statement.

**Property 3. Zero Offset.** If the closed-loop system converges to a steady state, it will converge to the desired steady state, even in the presence of modeling errors.

**Proof.** Application of the final value theorem to Eq. 23 yields the result.

Added to these properties is the fact that only two parameters,  $\alpha$  and  $\epsilon$ , need to be tuned, and their meaning is clearly understood; the lower their values, the faster is the system response and the lower is the robustness. Thus MDC is a powerful yet easy to implement controller design method.

## Examples

In order to demonstrate the features of MDC two simulation examples were run on a digital computer.

### Example 1

This example shows the power of MDC; it successfully controls a process that has time-delay and inverse response, and is open-loop unstable. The process is

$$\ddot{y}(t) + \dot{y}(t) - 2y(t) = \dot{u}(t - 0.1) - 2u(t - 0.1) + d(t) \quad (27)$$

or in the Laplace domain

$$\bar{y}(s) = \frac{(s-2)e^{-0.1s}}{(s+2)(s-1)}\bar{u}(s) + \frac{1}{(s+2)(s-1)}\bar{d}(s) \quad (28)$$

If a zero-order hold is used, the equivalent  $z$  domain representation (Stephanopoulos, 1984) is

$$\begin{aligned} \bar{y}(z) = & z^{-1-0.1/T} \frac{\left(1 - \frac{2}{3}e^{-2T} - \frac{1}{3}e^T\right) + \left(e^{-T} - \frac{1}{3}e^{-2T} - \frac{2}{3}e^T\right)z^{-1}}{(1 - e^{-2T}z^{-1})(1 - e^Tz^{-1})}\bar{u}(z) \\ & + z^{-1} \frac{\left(-\frac{1}{2} + \frac{1}{6}e^{-2T} + \frac{1}{3}e^T\right) + \left(-\frac{1}{2}e^{-T} + \frac{1}{3}e^{-2T} + \frac{1}{6}e^T\right)z^{-1}}{(1 - e^{-2T}z^{-1})(1 - e^Tz^{-1})}\bar{d}(z) \end{aligned} \quad (29)$$

where  $T$  is the sampling period. For  $T = 0.1$  this reduces to

$$A(z)\bar{y}(z) = B_+(z)B_-(z)\bar{u}(z) + C(z)\bar{d}(z) \quad (30)$$

where

$$A(z) = 1 - 1.9239z^{-1} + 0.9048z^{-2} \quad (31)$$

$$B_+(z) = z^{-2}(1 - 1.2222z^{-1}) \quad (32)$$

$$B_-(z) = 0.0858 \quad (33)$$

$$C(z) = z^{-1}(0.00485 + 0.00469z^{-1}) \quad (34)$$

If there is high confidence in the model, the choice  $\alpha = \epsilon = 0$  is appropriate. In this case Eq. 20 gives

$$P(z) = -40.607 + 32.133z^{-1} \quad (35)$$

$$Q(z) = 1 + 1.924z^{-1} + 43.404z^{-2} \quad (36)$$

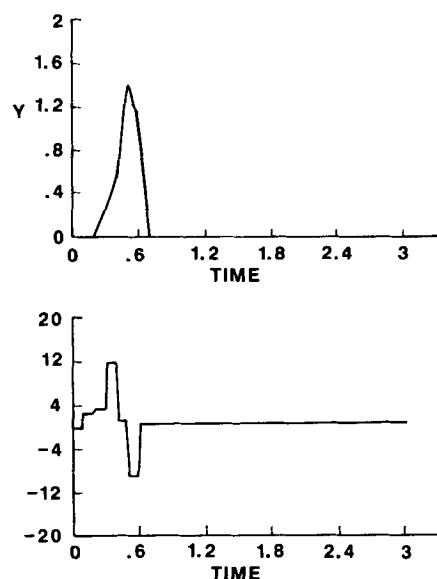


Figure 4. MDC response with  $\alpha = \epsilon = 0$  for the process of example 1.

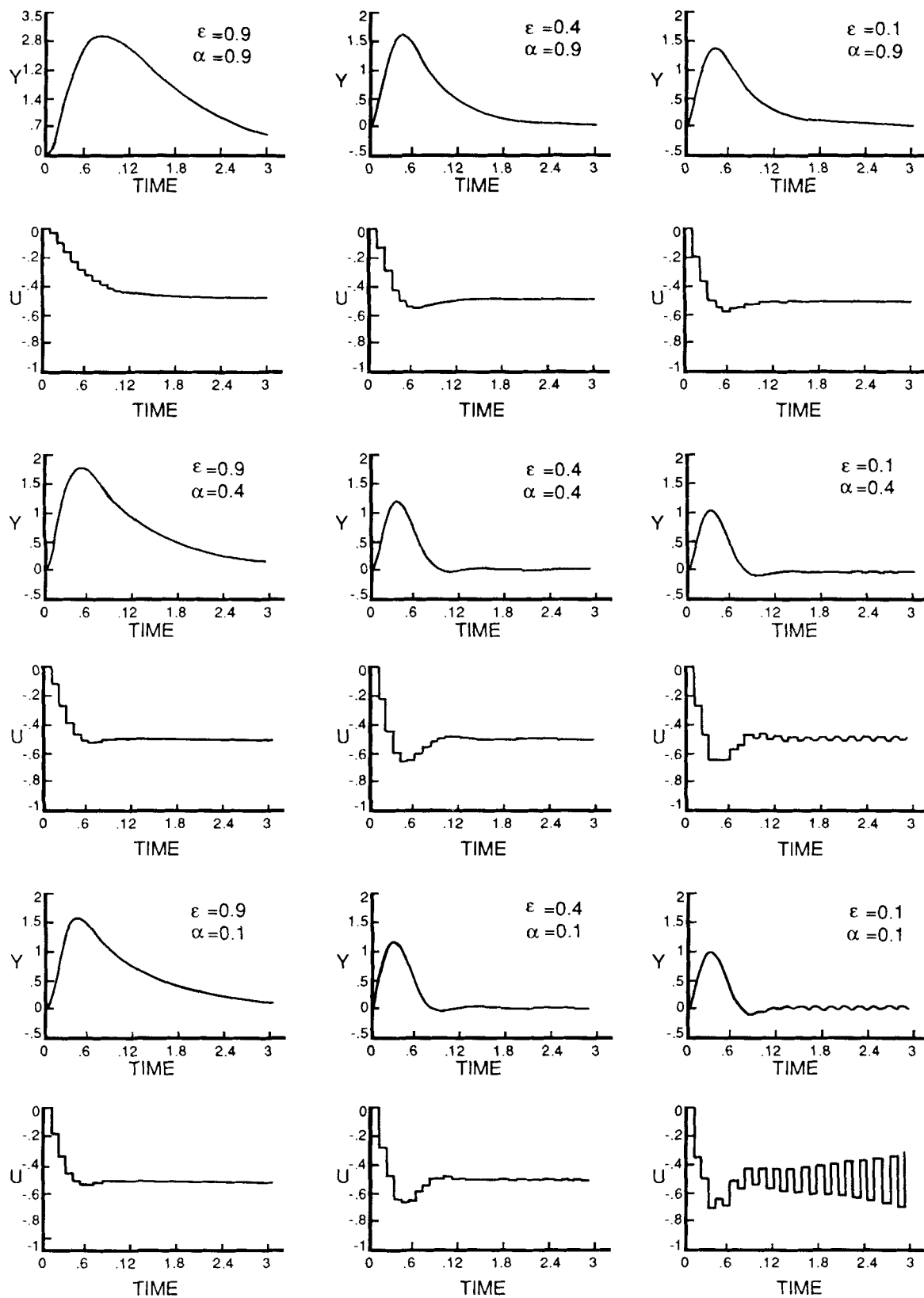


Figure 5. Dependence of MDC response on parameters  $\alpha$  and  $\epsilon$  for example 2.

and for filter  $F(z) = B_+(z)/B_+(1)$  the MDC law, Eq. 23, is

$$\begin{aligned} u(t) = & -52.454[0.0367u(t-T) + 0.913u(t-2T) \\ & + 0.06u(t-3T) + 3.522u(t-4T) \\ & - 4.551u(t-5T) - 9.024y(t) \\ & + 7.141y(t-T) - 40.607y(t-2T) \\ & + 81.763y(t-3T) - 39.273y(t-4T) + e(t)] \quad (37) \end{aligned}$$

Figure 4 presents the controller performance to disturbance changes. Initially the system was at the desired steady state  $y = u = d = 0$ . At time  $t = 0$  the disturbance changed to  $d = 1$ . It is seen that MDC rapidly returned the process to the desired steady state.

### Example 2

The example demonstrates the effect of the parameters  $\epsilon$  and  $\alpha$  on speed of response and robustness. Consider the second-order process:

$$\bar{y}(s) = \frac{2}{(s+0.1)(s+1)} \bar{u}(s) + \frac{1}{(s+0.1)(s+1)} \bar{d}(s) \quad (38)$$

Suppose that the exact model is not known. Instead, the model available has been obtained by fitting step change response curves with a first-order plus time-delay model (Stephanopoulos, 1984). Let this model be

$$\bar{y}(s) = \frac{20e^{-s}}{11s+1} \bar{u}(s) \quad (39)$$

For sampling period  $T = 1$  the equivalent  $z$  domain model is

$$A(z)\bar{y}(z) = B_+(z)B_-(z)\bar{u}(z) \quad (40)$$

where

$$A(z) = 1 - 0.913z^{-1} \quad (41)$$

$$B_+(z) = z^{-2} \quad (42)$$

$$B_-(z) = 1.74 \quad (43)$$

With the typical filter choice  $F(z) = (1 - \epsilon)z^{-2}/(1 - \epsilon z^{-1})$  the resulting MDC law is

$$\begin{aligned} u(t) = & [\epsilon - 0.913(1 - \alpha)]u(t-T) \\ & + [1 - \epsilon + 0.913(1 - \alpha)\epsilon]u(t-2T) \\ & + 0.913(1 - \alpha)(1 - \epsilon)u(t-3T) \\ & + \{-0.834(1 - \alpha)[y(t) - \epsilon y(t-T) - (1 - \epsilon)y(t-2T)] \\ & + (1 - \epsilon)[e(t) - 0.913\alpha e(t-1)]\}/1.74 \quad (44) \end{aligned}$$

Figure 5 presents the system response to a disturbance change from 0 to 1 (at  $t = 0$ ) for a range of values of  $\epsilon$  and  $\alpha$ . It is clearly seen that the higher the value of  $\epsilon$  or  $\alpha$  the more sluggish the response is and the more the system deviates from the desired steady state. However, due to the plant/model mismatch, very

low values lead to excessive oscillations and even instability, i.e., robustness is lost.

Figure 6 depicts the inversion-based IMC ( $\alpha = 1$ ) response for  $\epsilon = 0.1$ . This is about the lowest value that can be used; lower values lead to excessive oscillations and instability. It is seen that MDC with  $\epsilon = \alpha = 0.4$  takes care of the disturbance faster and with fewer oscillations.

### Conclusions and Significance

MDC effectively handles open-loop instability, inverse response, and time delays. It performs well both in following set points and in rejecting disturbances. Tuning is easy since only two parameters are involved and these directly control the trade-off between robustness and speed of response. These characteristics make MDC attractive for industrial applications.

### Notation

$A(z)$  = polynomial whose roots are the open-loop poles  
 $a_i$  = coefficients of  $A(z)$   
 $b_i$  = coefficients of  $B(z)$   
 $B_+(z)$  = factor of  $G_u(z)$  containing the noninvertible elements  
 $B_-(z)$  = numerator factor of  $G_u(z)$  containing the invertible elements  
 $C_i(z)$  = polynomial whose roots are the disturbance  $i$  open-loop zeroes  
 $c_{ij}$  = coefficients of  $C_i$   
 $d_i$  = unmeasured disturbance  
 $e$  = error  
 $F(z)$  = filter transfer function  
 $G(z)$  = polynomial whose roots are the closed-loop poles  
 $G_i(z)$  = disturbance pulse transfer function  
 $G_u(z)$  = plant pulse transfer function  
 $k$  = time delay in number of sampling periods  
 $n_x$  = order of polynomial  $X(z)$  in  $z^{-1}$  where  $X = A, P$ , or  $Q$   
 $n_+$  = order parameter of  $B_+(z)$ , Eq. 8  
 $n_-$  = order of polynomial  $B_-(z)$  in  $z^{-1}$   
 $n_i$  = order of polynomial  $C_i(z)$  in  $z^{-1}$   
 $s$  = Laplace domain independent variable  
 $t$  = time  
 $T$  = sampling period  
 $u$  = control variable  
 $U$  = generalized input  
 $y$  = measured variable

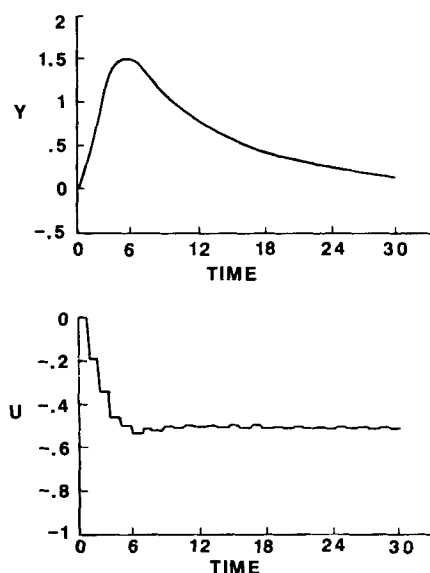


Figure 6. Inversion-based IMC response with  $\epsilon = 0.1$  for example 2.

$y_{sp}$  = set point  
 $z$  =  $z$  domain independent variable  
 $z_i$  = zeroes of  $G(z)$

### Greek letters

$\alpha$  = pole placement parameter  
 $\beta_i$  = coefficients of  $B_+(z)$   
 $\Delta_A$  = error in  $A$   
 $\Delta_B$  = error in  $B_+B_-$   
 $\epsilon$  = filter pole  
 $\lambda_i$  = open-loop pole

### Superscripts

$\cdot$  = time derivative  
 $-$  = Laplace or  $z$  transform  
 $^*$  = true process

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